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REVIEW ARTICLE

Applying Matrix Theory in Clark Hull's Drive Reduction Theory to analyze Learning Processes and Behavioral Responses in Educational Therapy for Students with Special Needs

Kok-Hwee, CHIA

Merlion Paediatric Therapy Clinic

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Author's email: dr.chia@merlionpaediatric.sg

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ABSTRACT

This article introduces Clark Hull's Drive Reduction Theory (DRT), where a drive acts as a reinforcement for a specific behavior of concern, through its linear 1×9 equation: $sEr = V \times D \times K \times J \times sHr - sIr - Ir - sOr - sLr$. This Hullian equation is further expanded by applying the Matrix Theory used in linear algebra within the domain of educational therapy. This 1×9 Hullian matrix serves as a structured approach to understand and address students' learning and behaviors by focusing on drives and rewards, building productive habits, and reducing barriers. Factorable causalities - autogenic, ecogenic, pedagenic, and psychosociogenic - also play their crucial roles in the drive states and learning processes that contribute to behavioral responses. With the integration of the Hullian DRT equation and Matrix Theory, educational therapists can systematically reduce behaviors driven by unmet needs, helping their students develop more adaptive responses and promoting a better learning environment.

Keywords: *Drive Reduction Theory, Clark Hull, Hullian Matrix*

1. INTRODUCTION

Clark Leonard Hull (b.1884–d.1952), an American engineer-turned psychologist, sought to explain learning and motivation by scientific laws of behavior (Miguez, Laborda, & Quezada-Scholz, 2022). After contracted poliomyelitis, he left his job as an engineer to pursue his psychological studies. In the later part of his life, Hull was recruited by James Rowland Angell, a former psychologist and was then the president of Yale University, where he conducted research studies to prove that his theories could predict behavior. His most influential works, *The Mathematico-Deductive Theory of Rote Learning* (Hull et al., 1940) and *Principles of Behavior* (Hull, 1943), established his framework of animal learning and conditioning as the leading theory of its time.

1.1 What is Drive Reduction Theory about?

Hull was well-known for his Drive Reduction Theory (DRT) that he had developed. According to Hull (1952), "the reduction of the drive acts as a reinforcement for that behavior *concerned*" (cited in Cherry, 2023, para. 10; the word in *italic* is added by the author). Hull created a mathematical equation (or formula) to explain his DRT of behavior as follows (see Cherry, 2023, para. 14-15):

$$sEr = V \times D \times K \times J \times sHr - sIr - Ir - sOr - sLr$$

where x is time (multiplication) operation and - is minus (subtraction) operation and also …

- *sEr:* Excitatory potential, or the likelihood that an organism will produce a response (*r*) to a stimulus (*s*)
- *V:* Stimulus intensity dynamism, meaning some stimuli will have greater influences than others
- *D:* Drive strength, determined by the amount of biological deprivation
- *K:* Incentive motivation, or the size or magnitude of the goal
- *J:* The delay before the organism is allowed to seek reinforcement
- *sHr:* Habit strength, established by the amount of previous conditioning
- *sIr:* Conditioned inhibition, caused by previous lack of reinforcement
- *Ir:* Reactive inhibition, or fatigue
- *sOr:* Random error
- *sLr:* Reaction threshold, or the smallest amount of reinforcement that will produce learning

This is a linear formula that can be framed in a *1 x 9* matrix (of one row of nine factors or entries in nine columns) to help educational therapists manage their students with learning and behavioral challenges. Hull's theory suggests that behaviors are driven by physiological needs (or 'drives') that create internal tension, which organisms (students) are motivated to reduce.

The framework of Hull's DRT has been applied in educational therapy, especially in the areas of diagnostic assessment and formative as well as summative evaluation of pre- and post-intervention results, such as in math problem-solving process and memory improvement.

2. MATRIX FRAMEWORK FOR HULLIAN DRIVE REDUCTION THEORY

In this article, the Matrix Theory (Franklin, 2012; Zhang, 2011) has been applied to explain Hull's theory of drive reduction. Below is a matrix (see in subsequent sections for Matrix Theory in this paper; also see Lyryx Learning & Kuttler, 2024, for detailed explanation) that breaks down Hull's equation/formula of a shorter version with two levels and its components in the context of an educational therapist managing a student (client):

First Level Equation:	$E = D \times H \times K$
Second Level Equation:	$R = E - I$
Combined Equation:	$R = [(D \times H \times K) - I]$

Each of the components or factors in the Hullian equation is further elaborated in Table 1 below.

Table 1. Description of Hullian Equation Terms

Component	Hullian Equation Term	Description	Application in Educational Therapy
Drive /D/	D	The internal state or motivation caused by an unmet need or tension.	Identify the student's (client's) primary drive(s) that are causing behavioral issues (e.g., need for attention, sensory regulation).
Habit Strength /H/	H	The strength of learned behaviors or habits in response to the drive.	Assess the habits or behaviors the client has learned in response to their drives (e.g., avoidance, aggression).
Incentive Motivation /K/	K	The external reward or stimulus strength associated with a response.	Implement a meaningful reward system to encourage adaptive behaviors (like token systems for positive actions).
Reaction Potential /E/	E = D x H x K	The likelihood of a behavior occurring, driven by the combination of drive, habit, and incentive.	Monitor and adjust interventions based on the likelihood of desired behaviors (if low, increase reward or decrease drive).
Inhibition /I/	I	The internal factors reducing the likelihood of a behavior occurring.	Address any potential obstacles that are preventing or suppressing positive responses (e.g., fatigue, sensory overload).
Response Probability /R/	R = E - I	The likelihood of a behavior after accounting for inhibition.	Use inhibition strategies to minimize maladaptive behaviors and reinforce desired behaviors in the client.

The explanation of how the above linear framework (see Table 1) is provided here to help understand of its application in educational therapy as follows:

1. Assessing Drive (*D*):
The educational therapist begins by identifying what drives the student's behaviors. For example, a student with a learning challenge may have a drive for sensory regulation, leading to behaviors like fidgeting or being distracted (e.g., watching trains passing by).

2. Evaluating Habit Strength (*H*):
The educational therapist identifies habits the student (client) has developed in response to the drive. For example, a student who avoids tasks due to frustration may show avoidance behaviors. The educational therapist can gradually teach more constructive behaviors by strengthening alternative habits through practice and encouragement.

3. Incentivizing with Motivation (*K*):
The educational therapist provides motivating incentives that reinforce desirable behaviors. In an educational context, this could mean using token rewards for staying on task or managing impulses, thus supporting the client in building positive habits.

4. Calculating Reaction Potential (*E*):
Using the equation $E = D \times H \times K$, the educational therapist can estimate the likelihood that a desired behavior will occur. If the reaction potential (E) is low, the educational therapist can increase the incentive (K) or address the drive (D) to improve behavior likelihood.

5. Applying Inhibition (*I*):
The educational therapist addresses inhibitory factors, such as over-stimulation or frustration, that reduce response probability. By minimizing these, the student can better focus on positive behaviors.

6. Outcome (*R*):
The educational therapist uses $R = E - I$ to gauge the likelihood of the target behavior after accounting for inhibitions. With this model, the educational therapist can iteratively adjust drive-reducing strategies and positive reinforcement.

This matrix (based on the Matrix Theory and linear algebra; see Lyryx Learning & Kuttler, 2024, for more detail) serves as a structured approach to understand and address students' behaviors by focusing on drives and rewards, building productive habits, and reducing barriers.

3. THE BASIC FORMULA IN HULLIAN THEORY

In DRT, behavior is motivated by biological drives and their reduction, where a need creates an internal drive that compels the organism (a human agent) to act in order to reduce that need. The basic formula in the Hullian theory is provided as follows:

$$sEr = sHr \times D \times K$$

where:
x (time) can also be substituted with a dot [·] as in $sEr = sHr \cdot D \cdot K$ and where …
- *sEr:* Excitatory Potential (likelihood of behavior) where /*s*/ is stimulus and /*r*/ is response
- *sHr:* Habit Strength (strength of learned behavior) where /*s*/ is stimulus and /*r*/ is response
- *D:* Drive (magnitude of need or motivation)
- *K:* Incentive (external reward or stimulus value)

To represent this in a matrix for a practical application, a matrix has to be constructed where rows represent different drives (motivations or needs) and columns represent factors that influence a student's behavior in response to these drives. This will help an educational therapist systematically analyze and address multiple motivational aspects.

3.1 Causalities of the Factors in Hullian Equation

In DRT, from the Hullian perspective, behavior is viewed as a response to drives, or internal states of tension, with learning and behavior resulting from an individual's efforts to reduce these drives. The linear Hullian equation (in its simplified form) can be expressed as follows:

$$sEr = V \times D \times K \times H - Ir - sIr$$

where:
- *sEr* is the excitatory potential or the likelihood of a behavior occurring
- *V* is the stimulus intensity dynamism (sensory stimuli)
- *D* is the drive (internal motivation or need)
- *K* is the incentive or external motivation
- *H* is the habit strength, representing learned behavior
- *Ir* is the reactive inhibition (physical and mental fatigue)
- *sIr* is the conditioned inhibition (suppression of a conditioned response)

In this simple version Hullian equation, various factorable causalities, which refer to situations where complex causes can be broken down or 'factored' (Merriam-Webster, 2024), into multiple contributing elements or factors (e.g., autogenic, ecogenic, pedagenic, and psychosociogenic), play roles in the drive states and learning processes that contribute to behavioral responses. In this concept, causality is not just a single, straightforward relationship (like A causing B), but rather a combination of various smaller, interconnected causes that collectively lead to a certain effect. For instance, in understanding the cause of a specific health issue, a factorable causality approach would look at multiple contributing factors, such as genetics, environment, lifestyle, and psychological aspects. By analyzing these factors separately and then examining how they interact, one can gain a clearer, more comprehensive view of the causative mechanisms.

Below is how each of these factorable causalities applies:

1. Autogenic Causality Factors (*AuC*):

These are internal, self-generated factors that originate within the individual. In Hullian terms, autogenic factors can contribute to the drive component (*D*) by affecting internal physiological or emotional states, such as hunger, thirst, or stress. For example, an individual's internal state of hunger (an autogenic factor) can drive them to learn behaviors related to seeking food.

2. Ecogenic Causality Factors (*EcC*):

These are external environmental factors that influence behavior. In Hull's equation, ecogenic factors relate primarily to the incentive value (*K*) since they represent the external stimuli or rewards that motivate behavior. For example, an inviting environment or access to resources can increase the likelihood of engaging in a particular behavior by enhancing its external appeal.

3. Pedagenic Causality Factors (*PeC*):

These are factors related to the developmental or experiential aspects of learning. Pedagenic factors align closely with habit strength (*H*), as they encompass past learning experiences and practice. For instance, a child repeatedly rewarded for completing homework builds a habit of doing it, reinforcing the likelihood of this behavior over time.

4. Psychosociogenic Causality Factors (*PSC*):

These include influences from social and psychological interactions, such as social norms, expectations, or pressures. These can influence any part of Hull's equation depending on context; for example, psychosociogenic factors can increase (*D*) if social pressures lead to feelings of anxiety, or they can boost (*K*) by attaching social reward to specific behaviors.

In addition, the categorization of reactive inhibition (*Ir*) and conditioned inhibition (*sIr*) as ecogenic and psychosociogenic respectively is grounded in the sources and mechanisms by which these inhibitions arise:

1. Reactive Inhibition (*Ir*):

This is typically associated with physical or mental fatigue resulting from repeated stimulation or activity, leading to a temporary decrease in response due to exhaustion. It is often regarded as ecogenic in nature because it arises in response to external stimuli or conditions (e.g., prolonged or repetitive tasks that induce fatigue). Though not directly caused by social or psychological factors, environmental demands play a significant role.

2. Conditioned Inhibition (*sIr*):

This involves learning that a specific stimulus predicts the absence of an unconditioned stimulus, resulting in the suppression of a conditioned response. Conditioned inhibition is generally considered psychosociogenic because it involves learning and adaptation within a social or psychological context. It reflects how experience, typically in an environment with specific cues, teaches an organism when not to respond.

Hence, */Ir/* aligns more with ecogenic causes due to external stimulation effects, while */sIr/* is more psychosociogenic because it arises from learned social or environmental cues.

In summary, within Hull's Drive Reduction framework, these causality factors together influence the learning process and behavioral responses by shaping how drives are reduced and habits are formed. Each of them can be understood as contributing to the components that drive and reinforce behavior:

- Autogenic causality factors (*AuC*) influence internal drive (*D*),
- Ecogenic causality factors (*EcC*) enhance external incentives (*K* and *Ir*),
- Pedagenic causality factors (*PeC*) build habit strength (*H*),
- Psychosociogenic causality factors (*PSC*) modulate drive and incentive via social and psychological contexts (*sIr*).

By incorporating the various causality factors in the Hullian equation, the following version has been created:

$$sEr = \{(sHr)\cdot(D)[(K)\cdot(J)\cdot(V)] - Ir\} - sIr$$

$$\downarrow$$

$$sEr \rightarrow \{Pedagenic_{+/-}(sHr) \cdot Autogenic_{+/-}(D) \cdot [Ecogenic_{+/-}(K) \cdot (J) \cdot (V)] - Ecogenic_{H+/L-}(Ir)\} - Psychosociogenic_{H+/L-}(sIr)$$

$$\downarrow$$

$$sEr = \{sHr_{(PeC)} \times D_{(AuC)} \times [K \times J \times V - Ir]_{(EcC)}\} - sIr_{(PSC)}$$

where

- +/- indicates positive or negative result in *H*, *D*, *K*, *J* and *V*
- *H+/L-* indicates positive for high (*H+* or ↑) or negative for low (*L- or* ↓) result in *Ir* and *sIr*

Results for *sEr* (Excitatory Potential) can be expressed in terms of +/-, H+/L-, ↑/↓, n/w (narrow/wide) in terms of degrees (°) of likelihood.

4. MATRIX REPRESENTATION OF HULLIAN DRIVE REDUCTION THEORY

For example, let us define a *4 x 4* matrix for simplicity, representing four main drives and four influencing factors:

Drives (*D*) are in the *i*-th rows (with the notation *ai*) and they represent the following needs:
1. Biological Needs (e.g., hunger, fatigue)
2. Safety Needs (e.g., fear, comfort)
3. Social Needs (e.g., acceptance, approval)
4. Achievement Needs (e.g., desire for success, mastery)

Factors (components involved in an equation) are in the *j*-th columns (with the notation *aj*):
1. Habit strength (*sHr*): the student's learned responses to satisfy the drive.
2. Drive intensity (*D*): the current magnitude or urgency of the drive.
3. Incentive Value (*K*): the perceived reward or positive outcome associated with behavior.
4. Observed Behavior (*sEr*): the resultant behavioral intensity or likelihood.

Hence, the matrix (*M*) can be structured as follows:

$$M = \begin{bmatrix} \text{Biological Needs} & sHr & D & K & sEr \\ \text{Safety Needs} & sHr & D & K & sEr \\ \text{Social Needs} & sHr & D & K & sEr \\ \text{Achievement Needs} & sHr & D & K & sEr \end{bmatrix}$$

5. APPLICATION OF MATRIX IN EDUCATIONAL THERAPY

Each entry or element in the matrix (as shown above) represents an interaction between a drive and a factor. By filling in values, an educational therapist could identify patterns and factors that increase or decrease specific drives for the student (client).

For instance, if a student shows a high value in *sEr* (Observed Behavior) for social needs, and this correlates with high values in *sHr* (habit) and *K* (incentive), the educational therapist might conclude that the student is particularly responsive to social rewards and might incorporate positive social reinforcement as part of behavior management.

In an example of this application with a student with behavioral challenges (e.g., impulsivity or distractibility), an educational therapist might take the following actions:

1. Identify dominant drives (*D*): Observe which drives are most active (e.g., high values in social or achievement needs).
2. Adjust incentives (*K*): Use higher */K/* values (incentives) in the matrix for */D/s* needing reinforcement.
3. Analyze habit strength (*sHr*): Assess the stability of */sHr/* for each */D/*. For */D/* showing low */sHr/*, build structured and repetitive interventions to reinforce desired behaviors.

This matrix allows educational therapists to track student responses across drives and behavioral influencers, making it easier to personalize strategies and manage the student's unique learning and behavioral needs effectively.

6. MEASUREMENTS FOR HULLIAN EQUATION FOR LEARNING AND BEHAVIOR

To measure each component in the Hullian equation for learning and behavior, specific tests need to be taken into consideration to evaluate the factors contributing to the behavior and learning potential (Excitatory Potential, or *sEr*). For instance, two suggested assessments for each component are provided as follows:

1. Habit strength (*sHr*)
   - Observation-Based Habit Consistency Rating (e.g., Ekma et al., 2020): Track the consistency and frequency of a specific learned behavior over time through structured observation, scoring each instance to determine habitual performance.
   - Frequency of Repetition Test (e.g., Orth et al., 2015): Measure how often a behavior is repeated within set intervals, which helps determine how well-established the behavior is as a habit.

2. Drive (*D*)
   - Motivation Assessment Scale (e.g., Ray-Subramanian, 2013): Gauge the intensity of internal motivations driving specific behaviors, like engaging with a task or responding to an intervention, by measuring persistence and focus.
   - Behavioral Persistence Test (e.g., Oeri, 2022): Assess the time or effort the individual continues a behavior in the face of difficulty, as a higher drive often correlates with increased persistence.

3. Incentive motivation (*K*)

- Preference Assessment (see How-to-ABA, 2023): Present various reinforcers to determine their effectiveness in motivating the individual, highlighting which incentives are most motivating.
- Token Economy Response Rate (Ivy et al., 2017): Track the rate of response when using token rewards, examining if higher incentives (e.g., more tokens) produce stronger response rates.

4. Lapse between Sessions (*J*)
   - Memory Retention Test (e.g., Sandberg, 2011; Williams, 1990): Assess retention of learned behaviors or skills after different intervals, providing insight into how the lapse between sessions affects learning maintenance.
   - Delayed Recall Task (Camos & Portrat, 2015): Examine how the individual recalls information or skills after different time delays, indicating how session frequency might influence learning stability.

5. Stimulus intensity dynamism (*V*)
   - Sensory Processing Measure (SPM) (Brown, Almiento, Yu, & Bhopti, 2023; Parham et al., 2021): Evaluate the response to varying levels of sensory input (e.g., loud vs. soft sounds), observing how intensity impacts engagement.
   - Reaction Time Test to Stimuli (e.g.,Pascual-Leone et al., 1992; Pins & Bonnet, 1996): Measure the speed of response to varying intensities of stimuli to assess how quickly a person engages with more intense versus less intense stimuli.

6. Reactive inhibition (*Ir*)
   - Fatigue Index Assessment (e.g., Belza et al., 2018; Shele et al., 2014): Track the decrease in task performance or response rate over time to measure how reactive inhibition (or mental fatigue) affects persistence.
   - Interruption Task Test (e.g., Powers & Scerbo, 2023; Wu, Gao, & Liu, 2023): Assess how well the individual resumes a task after being interrupted, revealing sensitivity to reactive inhibition effects on task continuation.

7. Conditioned inhibition (*sIr*)
   - Extinction Task (e.g., Huang et al., 2025; Klein et al., 2023): Observe how well an individual can stop a learned behavior when reinforcement is withheld, indicating the degree of conditioned inhibition.
   - Negative Reinforcement Response Test (e.g., Koob, 2021; Morse & Kelleher, 2022): Evaluate response levels when a previously positive stimulus is removed or paired with a negative outcome, assessing resistance to response.

7. APPLICATION OF THE HULLIAN EQUATION IN ITP/IEP CONTEXT

In an Individualized Treatment Plan (ITP) or Individualized Education Program (IEP), each Hullian component can target a specific aspect of learning and behavior modification. The tests can provide baseline measures and ongoing assessments for setting objectives (e.g., strengthening habit formation, adjusting incentives, managing reactive inhibition). This systematic approach can allow educational therapists (ETs) to align interventions with specific Hullian factors, improving the effectiveness of tailored strategies in response to the student's unique learning needs.

7.1 What is Matrix Theory in Linear Algebra?

Matrix Theory is a fundamental area in linear algebra (Franklin, 2012; Lyryx Learning & Kuttler, 2024; Zhang, 2011), dealing with matrices (symbolized by uppercase letters): rectangular arrays of numbers, symbols, or expressions (known as entries or elements, symbolized by lowercase letters, with two subscript indices) arranged in *i*-th rows and *j*-th columns. The *m* x *n* (or *m-by-n*) matrices are powerful tools for organizing and analyzing data, as they allow for the representation of complex systems, transformations, and relationships in a structured way. Operations on matrices (e.g., addition, multiplication, and finding determinants and inverses) enable the solution of linear equations (Lang, 2002), which are essential in various applications across mathematics, physics, computer science, and engineering.

A matrix over a field (*F*) is simply a matrix where all the entries come from a particular field (*F*) (Fraleigh, 1976; Nering, 1970). A field is a set equipped with two operations (addition and multiplication) that satisfy certain properties, such as having additive and multiplicative identities (0 and 1, respectively), and every non-zero element has a multiplicative inverse. Common examples of fields are provided here: (i) The field of real numbers *{R}*; (ii) The field of complex numbers *{C}*; (iii) The field of rational numbers *{Q}*; (iv) Finite fields like *{Z}/p* or *{Z}*, where */p/* is a prime number (integers modulo *p*).

In other words, when a matrix is said to be over a field (*F*), it means each entry in the matrix is an element of */F/*, and matrix operations (e.g., addition and multiplication; see Lang, 2002, for detail) are performed according to the rules of */F/*. For example, a matrix with real entries would be a matrix over *{R}*, and a matrix with entries modulo 7 would be a matrix over *{Z}/7* or *{Z}*.

7.2 Application of Matrix Theory in Educational Therapy

In educational therapy, the Matrix Theory (Franklin, 2012; Zhang, 2011) can be a framework for diagnostic assessment and follow-up intervention. Briefly described, in terms of a diagnostic assessment conducted during the educational therapy, matrices can represent data from various assessment tools, allowing therapists to systematically record a student's responses or performance across multiple skills or tasks. For instance, each row (*ai*; also known as row vector) in the matrix might represent a different session or assessment instance, while each column (*aj*; also known as column vector) in the matrix could represent specific skills or areas being measured, like attention span, cognitive processing, or motor skills. This organization aids in identifying patterns and areas that need intervention. As for the follow-up intervention, after the initial assessment, matrix-based tracking can help educational therapists monitor a student's progress across different skills over time. By analyzing changes in *m* x *n* matrix data, educational therapists can determine the effectiveness of interventions, adjust strategies, and target specific areas for improvement. For example, they might use *m* x *n* matrix operations to evaluate growth in certain skills or compare results across different intervention phases.

Below are seven examples of how the Matrix Theory in linear algebra (Lyryx Learning & Kuttler, 2024) can be adapted and applied to both diagnostic assessment and intervention strategies in educational therapy for children with autism and dyslexia. Each of these applications uses Matrix Theory (see Franklin, 2012; Zhang, 2011) to structure, analyze, and interpret data to tailor educational interventions, helping to meet the individual needs of children with autism or dyslexia in a more structured, quantitative way.

Examples:
1. Behavioral data analysis and pattern recognition
- Application: Observing patterns in behavioral data (such as eye contact, task engagement, or response time) is essential in understanding a child's progress and needs. By organizing this data in matrices, educational therapists can identify recurring patterns.
- Case Use:
  (a) For a child with autism, matrices can reveal social behaviors over multiple sessions.
  (b) For a child with dyslexia, they can help track reading speed and accuracy across various tasks.

2. Multi-sensory integration assessment

- Application: Children with autism or dyslexia often struggle with sensory integration. By using matrices to categorize and quantify responses to sensory inputs, educational therapists can assess and modify multi-sensory integration interventions.
- Case Use: Matrices can track a child's sensory responses (visual, auditory, tactile) and help in adjusting intervention strategies based on responses to each type of sensory input.

3. Tracking Progress in Skill Acquisition

- Application: Skill acquisition (e.g., learning phonics or social cues) can be tracked using matrices where rows represent sessions and columns represent specific skills. This matrix can highlight areas where progress is slower or faster than expected.
- Case Use:
  (a) For dyslexia, matrices help evaluate progress in phonological awareness.
  (b) For autism, they track social skills, like appropriate responses in social situations.

4. Error pattern analysis

- Application: Error matrices are valuable in analyzing types and frequencies of errors in a child's responses, such as misreading words or missing social cues. By quantifying errors, educational therapists can target specific areas.
- Case Use:
  (a) In dyslexia, matrices can show consistent reading errors (e.g., misinterpreting certain phonemes).
  (b) For autism, matrices could help track nonverbal cues that the child fails to pick up in social contexts.

5. Visual processing and spatial mapping

- Application: Matrix transformations help evaluate a child's spatial processing skills by analyzing tasks that involve visual and spatial elements, such as block designs or pattern completions.
- Case Use:
  (a) For autism, matrices can be used to analyze performance in visual-spatial tasks to support visual processing.
  (b) For dyslexia, matrices assist in identifying issues with spatial orientation of letters.

6. Modeling intervention impact across time and domains

- Application: Educational therapists can use matrices to model intervention outcomes across different sessions, input types (like verbal, visual, or kinesthetic), and performance metrics, creating a time-series of results to gauge intervention impact.
- Case Use:
  (a) For dyslexia, matrices might show improvement in reading comprehension over time.
  (b) In autism therapy, matrices can model social responsiveness across contexts (classroom vs. therapy room).

7. Cognitive load analysis for task sequencing

- Application: By organizing tasks in a matrix according to difficulty and cognitive demands, educational therapists can create optimal task sequences for minimizing cognitive overload.
- Case Use:
  (a) In dyslexia therapy, matrices can help determine the best sequence of tasks based on phoneme difficulty.
  (b) For autism, matrices assist in sequencing social interactions from simpler to more complex, based on the child's comfort and skill levels.

Overall, the Matrix Theory (Franklin, 2012; Zhang, 2011) provides a structured approach for both assessment and intervention in educational therapy, enabling data-driven decisions that can better support individual learning needs of students with special needs.

8. UNDERSTANDING AND MANAGING MOTIVATION IN STUDENTS WITH SPECIAL NEEDS

In many cases encountered by the educational therapists, one key issue of concern is related to the motivation of students with special needs (see Martin, 2021; Montgomery, Montgomery, & Montgomery, 2023). Ouherrou, Benmarrakchi, Elhammoun, and El Kafi (2018) in their study reported that “[S]uccess or failure of children with Special Educational Needs (SEN) depends on their motivational sate. Motivation can affect children performance of learning skills, strategies and behaviors. Children with SEN suffer from emotional problems related to their learning difficulties such as depression and anxiety” (para. 1). It is always this group of lowly motivated and unmotivated students being referred to school counselors for support and guidance (Kahveci, 2016; Tarver-Behring & Spgna, 2004; Tarver-Behring, Spagna, & Sullivan, 1998).

Hull’s Drive Reduction Theory posits that motivation arises from physiological needs that create a state of tension or ‘drive.’ Behavior is directed towards actions that reduce these drives, thereby restoring homeostasis (Hull, 1952). In educational therapy, this theory of drive reduction can be applied to understand and manage behaviors that stem from unmet needs or motivations.

To model this with matrices in a simplified linear form, a matrix representing the various ‘drives’ (*D*), ‘responses’ ®, and ‘habits’ (*H*) can be easily created. For educational therapists, these elements can be mapped to behavioral interventions to help their students (clients) with learning or behavioral issues by reducing disruptive drives. The author of this paper has suggested the following steps to be taken:

Step 1: Define the Drives, Responses, and Reinforcement

1.1. Drives (*D*): These represent underlying needs or triggers for behavior, such as attention-seeking, sensory stimulation, or escape from a demand.
1.2. Responses (*R*): Possible actions the client might take in response to drives (e.g., task completion, disruptive behavior).
1.3. Reinforcement (*H*): This factor shows the likelihood of a response being chosen in the future, based on the reduction of the drive it provides.

Step 2: Matrix Representation of the Hullian Equation

Hull’s equation can be simplified to focus on how drives influence response probability. A simplified version of the Hullian equation can be represented as follows:

$$B = D \cdot R \cdot H$$

where:

- *B* represents the strength of a behavior.
- *D* is a matrix of drives with weights representing the intensity of each drive.
- *R* is a matrix of possible responses with weights indicating their likelihood given the drives.
- *H* represents reinforcement, showing the habit strength of each response to reduce the drives.

Below is a sample matrix setup for educational therapy. Assume there are three main drives (e.g., attention-seeking, sensory-seeking, escape from demand) and three responses (e.g., engaging in a task, disruptive behavior, seeking assistance).

1. Drive Matrix (D):

$$D = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$$

2. Response Matrix (R):

$$R = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.7 & 0.4 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}$$

3. Reinforcement Matrix (H):

$$H = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.4 & 1 & 0.6 \\ 0.3 & 0.4 & 1 \end{bmatrix}$$

How the above three matrices work is explained as follows in this example of the model:

1. Multiply (x) */D/*, */R/*, and */H/* matrices: The product of these matrices will yield a matrix */B/* that indicates the strength of each behavior based on the current drives, possible responses, and reinforcement strength (as shown in Table 2 below).

Table 2. Multiplication of D x R x H Matrices = B Matrix

Drive	Response	Reinforcement	Behavior Strength
$D = \begin{bmatrix} 0.8 & 0 & 0 \\ 0 & 0.6 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}$	$R = \begin{bmatrix} 0.5 & 0.2 & 0.3 \\ 0.3 & 0.7 & 0.4 \\ 0.4 & 0.3 & 0.5 \end{bmatrix}$	$H = \begin{bmatrix} 1 & 0.5 & 0.2 \\ 0.4 & 1 & 0.6 \\ 0.3 & 0.4 & 1 \end{bmatrix}$	$B = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.42 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}$

2. Interpret results for behavior management: The resulting values in */B/* can show the educational therapist which responses are likely to reduce specific drives. For instance, if sensory-seeking behaviors have high drive (value in */D/* and specific interventions (like providing sensory activities) have high reinforcement (value in */H/*), this approach should reduce disruptive sensory-seeking behavior.

3. Application in educational therapy: By analyzing the */B/* matrix, an educational therapist can identify which drives (e.g., escape or attention-seeking) are motivating behaviors. Interventions can be designed to either provide alternative responses (redirect the drive) or reduce the drive through reinforcement (like rewards for task completion).

The /*B*/ (Behavior Strength) matrix represents the outcome of multiplying the three matrices: /*D*/ (Drive), /*R*/ (Response), and /*H*/ (Reinforcement). The resulting /*B*/ matrix shows how these three factors interact to influence behavior, with each element reflecting the strength of behavior resulting from the combination of these factors.

Structure of /B/ matrix:

$$B = \begin{bmatrix} 0.4 & 0 & 0 \\ 0 & 0.42 & 0 \\ 0 & 0 & 0.45 \end{bmatrix}$$

This matrix is a diagonal matrix, where:

- The diagonal entries are 0.4, 0.42 and 0.45.
- All off-diagonal entries are 0, indicating no interaction between different components in those cases.

From the diagonal matrix as seen above, its results can be interpreted as follows:

1. Diagonal entries: The diagonal elements [0.4, 0.42, 0.45] represent the strength of behavior (*B*) for each specific combination of drive (*D*), response (*R*), and reinforcement (*H*). Specifically, these values correspond to how these factors come together in the different components or dimensions of the behavior system. Each of these values represents a behavior strength score, reflecting different conditions or types of behavior based on:

   - The diagonal entries are 0.4, 0.42 and 0.45.
   - Drive (*D*): The motivation or need that drives the behavior.
   - Response (*R*): The actual behavior or reaction produced in response to the drive.
   - Reinforcement (*H*): The consequences or outcomes that either strengthen or weaken the behavior.

   The behavior strength (*B*) values increase from 0.4 to 0.42 to 0.45, suggesting that, in the particular context an educational therapist is examining, behavior strength (*B*) tends to increase slightly as they move from one combination of these factors to another.

2. Off-diagonal zeros: The fact that all the off-diagonal elements are zero means that there is no interaction or cross-influence between the different components. Specifically:

   - The Drive (*D*) and Response (*R*) combinations do not interact with each other in terms of their effect on
     behavior strength.
   - Similarly, */D/* and */R/*, as well as */R/* and */H/*, do not directly influence each other in this model. This suggests the simplified model of behavior where the effects of drive (*D*), response (*R*), and reinforcement (*H*) on behavior strength (*B*) are considered independently from one another.

3. General interpretation of the results of */B/* matrix is as follows:
   - The values [0.4, 0.42, 0.45] might indicate that for each factor (Drive, Response, Reinforcement), the strength of behavior (*B*) is positively influenced, but in a moderate manner. These values represent mild to moderate behavior strength outcomes, where reinforcement (*H*) might have a slightly greater impact than drive (*D*) or response (*R*), given that the third entry (0.45) is the largest.
   - Since all off-diagonal entries are zero, this model could be used to represent a linear, additive approach to understanding behavior, where the factors do not interact or compound one another.

9. CONCLUSION

In conclusion, the */B/* matrix tells the educational therapists that the behavior strength (*B*), as a result of the interplay of drive (*D*), response (*R*), and reinforcement (*H*), is moderately influenced by each factor independently. The diagonal values represent the strength of behavior in different conditions, with reinforcement (*H*) appearing to have the strongest influence (0.45). However, there is no interaction between the components (Drive, Response, Reinforcement), suggesting a simplified model of behavior where these factors do not combine or influence each other directly.

By applying this structured approach involving the Hullian equation of DRT and the Matrix Theory, educational therapists can systematically reduce behaviors driven by unmet needs, helping their students (clients) develop more adaptive responses and promoting a better learning environment.

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11. COMPETING INTERESTS

The author has declared that no competing interests exist.

12. FINANCIAL DISCLOSURE

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13. ARTIFICIAL INTELLIGENCE DISCLOSURE

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